The Jarque-Bera test

It is a measure of departure from normality, based on the sample skewness and kurtosis. The test statistic is defined as

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right),$$

where *n* is the number of observations; *S* is the sample skewness, and *K* is the sample kurtosis, defined as

$$S = \frac{\mu_3}{\sigma^3} = \frac{\mu_3}{\left(\sigma^2\right)^{3/2}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x-\bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x-\bar{x})^2\right)^{3/2}}$$

$$K = \frac{\mu_4}{\sigma^4} = \frac{\mu_4}{\left(\sigma^2\right)^{2}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x-\bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x-\bar{x})^2\right)^{2}}$$

where μ_3 and μ_4 are the third and fourth central moments, respectively, $\bar{x}$ is the sample mean, and σ^2 is the second central moment, the variance.

Hypothesis testing:

The null hypothesis is that the data are from a normal distribution, in the form of a joint hypothesis that both the skewness and excess kurtosis (*K*-3) are 0, since samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 0. As the definition of *JB* shows, any deviation from this increases the JB statistic.

The statistic *JB* has an asymptotic chi-square distribution with two degrees of freedom.

In practice:

For a chosen significance level of 5%, the critical value of the chi-square distribution is approximately 6. Therefore, whenever the calculated statistic exceeds this value, we can conclude that in 95% of the cases the distribution considered is either asymmetric, or has higher tails than a normal distribution, or both.