The Omega Measure

The Omega measure was introduced by Keating and Shadwick in 2002 as an alternative to mean-variance or Value at Risk measures. The Omega function captures all the higher moment information in the returns distribution, regardless of the shape of the distribution and the risk preferences of the investor. It requires however the exogenous specification of a level of return against which a given outcome will be viewed as a gain or a loss.

This loss threshold represents the investor's acceptable level of return, below which any return is regarded as a loss. At this pre-specified level, Omega can be used to rank portfolios unequivocally. The rankings will differ in general from those of traditional measures such as the Sharpe Ratio. For any threshold, Omega is simply computed as the ratio between the sum of probability weighted returns above threshold and the sum of probability weighted returns below the threshold.

Properties of the measure:

- It is mathematically equivalent to the distribution of returns.
- It is a monotone decreasing function on the returns space, and it takes values from $+\infty$ to 0.
- It takes the value 1 at the mean of the distribution.
- The higher the mean, the higher the omega, all else being constant.
- The higher the variance, the flatter the slope of the omega function. If all else is constant, this implies that there will be one reversion of preferences dictated by Omega over the returns space. The higher variance fund will be preferred at higher threshold levels.
- Skewness and higher odd moments: omega is higher for positive values.
- Kurtosis and higher even moments induce an odd number of crossings between the omega function plots, and thus several changes of rankings along the returns distribution.

Uses of the measure:

- Kazemi, Schneeweis, and Gupta (2003) showed that Omega is equivalent to the ratio between the prices of a European call and a European put on the investment, with the same strike price. They also propose a Sharpe-Omega ratio which gives the same ranking as the Omega function for any given threshold *L*:
$$Sharpe - Omega = \frac{\bar{x} - L}{P(L)}$$
- The variation of Omega over time for one stock serves as indicator of the manager's skills (we have it in the Rolling Omega chart).
- Keating and Shadwick mention that it can be used in portfolio optimization, but I need to read more on the topic.

A final observation concerns our Pack Hedge quant reports: the Omega chart displays only a range of the returns distribution. This has limited usefulness, in the sense that our investors' loss threshold may lie in this range or not.

Appendix: Omega in practice

I have calculated the Omega functions for three of our screened funds: Horseman Global Macro, Bear Stearns Emerging Markets, and Carrousel, over the same period of time: from Oct.2002 to Aug.2007 (59 observations). They display the following statistics, and the histograms have quite different shapes, which is useful for illustrative purposes.

	Horseman	Bear Stearns	Carrousel
Mean	1.51%	2.05%	0.87%
Median	1.63%	1.89%	0.92%
Standard Deviation	4.26%	4.36%	1.22%
Skewness	-0.22	-1.28	-0.62
Excess Kurtosis	-0.46	4.27	0.82

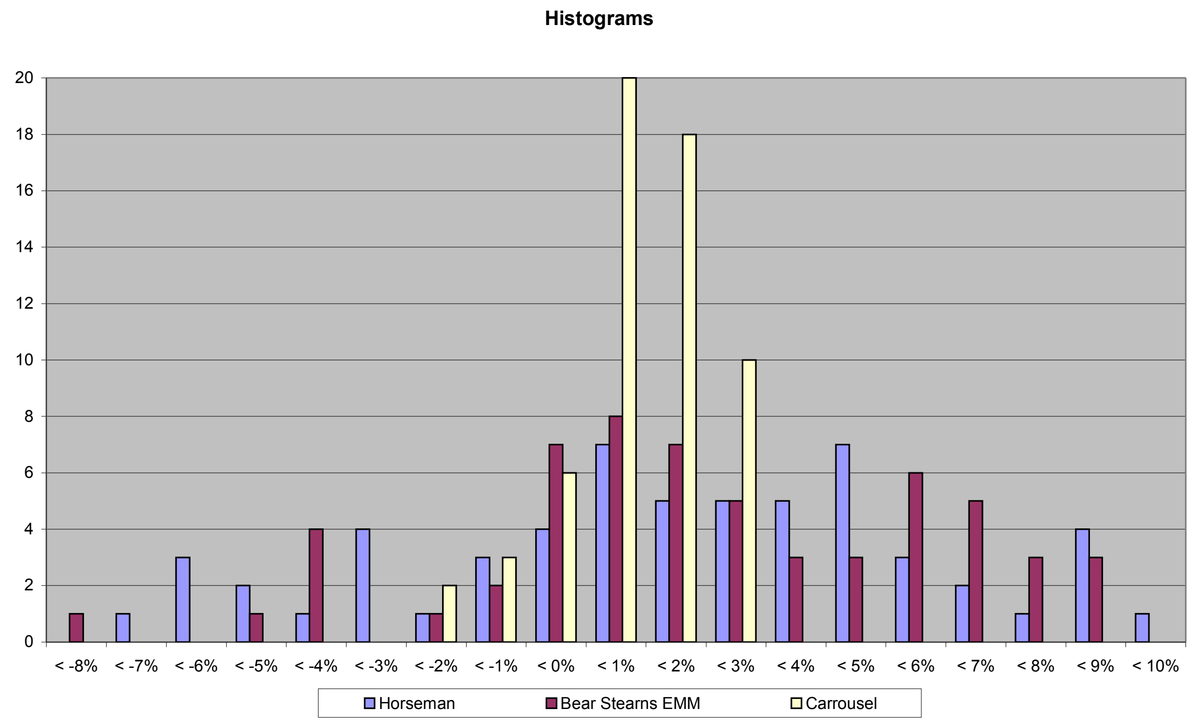

Figure 1: The histograms have quite different shapes.

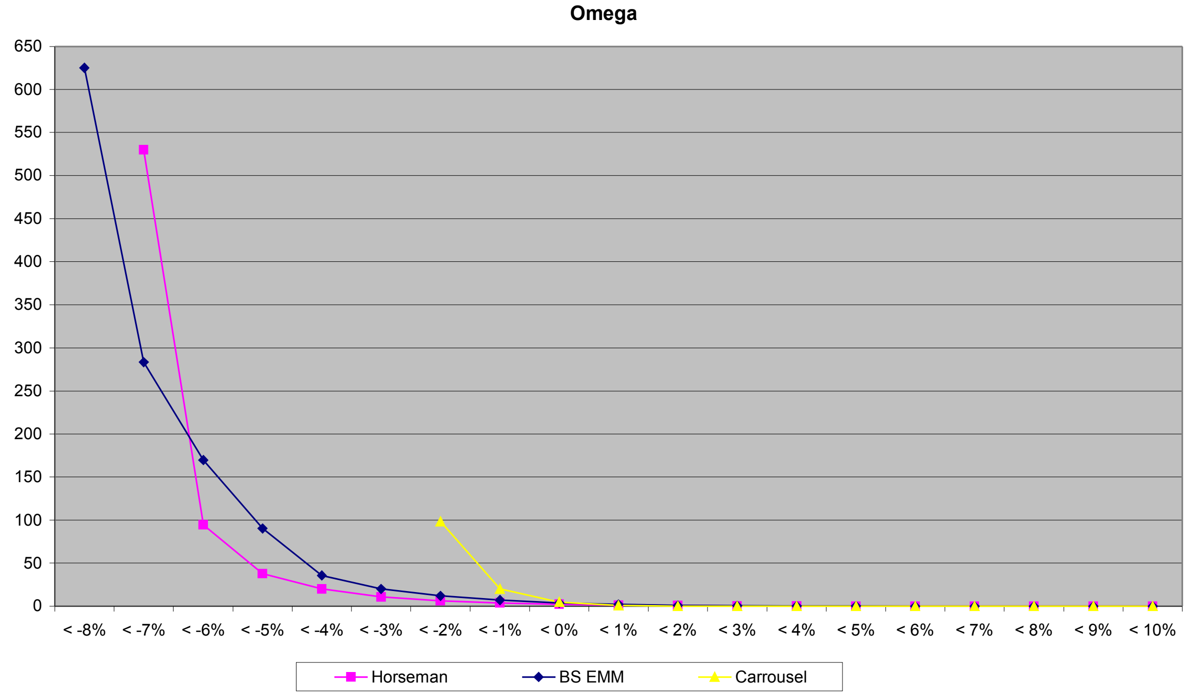

Figure 2: Omega functions.
Bear Stearns has the highest mean and volatility, while Carrousel the lowest mean and volatility. All are negatively skewed, but Bear Stearns is the most negatively skewed among them. Horseman has the lowest kurtosis, and Bear Stearns the highest.

A much more useful chart in terms of visibility is the plot of Omega natural logarithms, which preserves the Omega rankings:

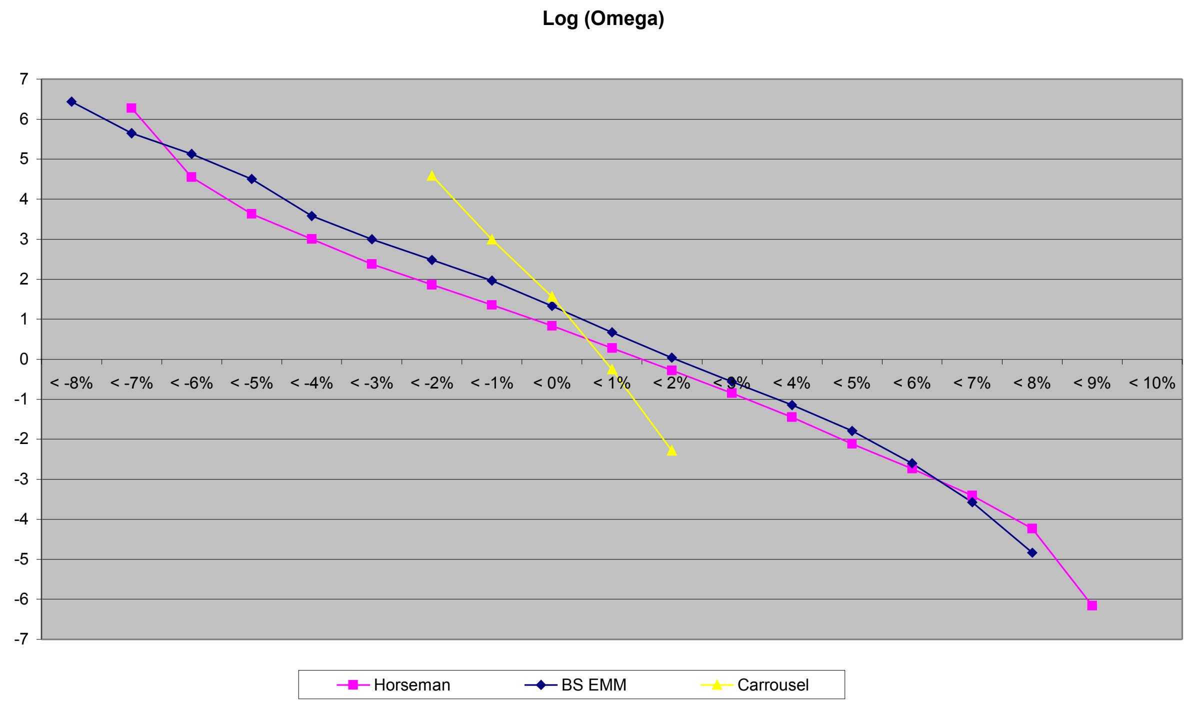

Figure 3: Logarithmic Omega.
Carrousel displays the steepest curve, having the lowest volatility. As predicted, Carrousel is preferred to the other two funds only at low thresholds.
Bear Stearns is preferred to Horseman at all levels except the lower tail, because it has a larger mean. At the tails, Horseman is preferred to Bear Stearns due to its lower kurtosis.